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# Solution of Fuzzy Assignment Problem Hungarian Algorithm and Using Branch and Bound Method

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**Abstract:** The present study develops approaches to solve assignment problems and FUZZY assignment problems using branch and bound methods. Assignment problem is a fundamental optimization problem in which tasks must be assigned to resources in such a way that the total cost or time is minimized or the total profit is maximized. Fuzzy assignment problems where the profit is not deterministic number but imprecise one. The profit matrix of the assignment problem are triangular fuzzy numbers, and optimal solution can be obtained using Hungarian algorithm and branch and bound method converting fuzzy numbers into crisp number. The efficiency of the proposed method is illustrated by a numerical example.

**Keywords:** Hungarian method, fuzzy numbers, triangular fuzzy numbers, crisp numbers, branch and bound method of an assignment problem

## 1. Introduction:

In operation research, the Assignment subject (AP) is a traditional and frequently researched subject. It is an instance of the Transportation Problem (TP) in particular. Issues with assignments occur in many different domains, including healthcare, transportation, education, sports, and work assignments. Within the fields of operations research or optimization, combinatorial optimization issues have extensively researched this subject (Basirzadeh, H. (2012). Numerous issues have been resolved globally by using assignment problems. Around the world, this issue has been frequently considered in numerous educational settings. An assignment issue is an investigation of the optimal strategy to assign things to objects .The assignment aims to provide underlying combinatorial structure, while the objective function represents the desire for maximum optimization(Wulan , E.R. et al. (2020)). Numerous different approaches have been put forth , including the hybrid algorithm , the local search-based the population search-based , the exact technique , the heuristics method and the local search-based .Votaw and Orden's linear programming technique , the transportation algorithm, or Kuhn's Hungarian method can all be used to solve the Assignment Problem(Burkard R. et al 2012). The standard assignment problem was initially solved using the Hungarian approach, which is recognized as the first applied method. A labeling approach for resolving the transportation and assignment difficulties was presented by Balinski and Gomory (Lin C.J.et al 2004 ).In order to solve fuzzy assignment problems, Nagarajan and Solairaju presented a method that employs the Robust ranking technique with fixed fuzzy numbers (Kalaiarasi K. et al 2014). The multi-criteria problem as a cost time assignment problem. Many academics have created various methodologies for resolving generalized assignment problems in the literature that is currently available.

The main objective of assignment problem which is also a special type of linear programming problem is to obtain the optimum assignment for the number of tasks (Jobs) that is equal to the number of resources (workers) at a minimum cost or maximum profit. This place and vital role in assigning people to jobs, classes to rooms, operators to machines etc. An optimal solution to assignment problems can be achieved using various algorithms such as linear programming , Hungarian algorithm, etc. Hungarian method and branch and bound techniques, help to optimize assignments under uncertainty . This study explores the concepts, methods and applications of fuzzy assignment problem highlighting its advantages.

## 2. Definitions

### 2.1. Assignment Problem

The Assignment problem is defined as minimization Linear programming problem. Also, Assignment problem is expressed in mathematical form:

$$Max\ Z = \sum\sum cijxij \quad (1)$$

$$i \in Aj \in T$$

Subject to the constraint:

$$\sum xij = 1, for\ j = 1, \ldots., n \quad (2)$$

$$j \in T$$

$$\sum xij = 1, for\ j = 1, \ldots., n \quad (3)$$

$$i \in A$$

$$xij \geq 0 \quad (4)$$

The assignment problem is a fundamentals combination optimization issue. This to different objects while minimizing the cost.

**2.2 Lower bound**

The lowest cost of distribution work in assignment problems is known as the lower bound. The ultimate or ideal outcome cannot be less than the lower bound.

**2.3. Branch and bound**

A technique called the branch, and bound method is based on listing all potential answers to a combination optimization problem. It is employed to locate the best solution in general, discrete, and combination mathematics. To tackle optimization problems, the concepts of trees, logic trees, and boundaries are used.

The Assignment Problem of n×n cost matrix of real numbers is as follows. Consequently, the general assignment problem has a mathematical form.

$$Max\ Z = \sum\sum cijxij$$

$$i \in 1j \in 1$$

Subject to the constraint:

$$\sum xi = 1\ i = 1$$

$$\sum xj = 1\ j = 1$$

$$xii = \begin{Bmatrix} 1 \text{ if jth job is askigned to the } i^{\text{th}} \text{ person} \\ 0 \text{ Otherwise} \end{Bmatrix}$$

**2.4. Fuzzy set:**

A fuzzy set is characterized by a membership function mapping elements of a domain , spaces or the universe of discourse X to the unit interval [0,1]

$$(ie)A\{x, \mu(x); xX\}$$

**2.5. Fuzzy number**

A real fuzzy number ā is a fuzzy subset of the real number R with membership function $\mu_a$ satisfying the following conditions,

i. Is continuous from R to the closed interval [0,1]
ii. Is strictly increasing and continuous on$[a_1, a_2]$
iii. Is strictly decreasing and continuous on$[a_3, a_4]$

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**2.6. Arithmetic Operations of Triangular Fuzzy Numbers:**

Let $A = (a1, a2, a3)$ and $B = (b1, b2, b3)$ be two triangular fuzzy numbers. Then

1. $(a1, a2, a3)\ (b1, b2, b3)\ =\ (a1 + b1, a2 + b2, a3 + b3)$
2. $k(a1, a2, a3) =\ (ka1, ka2, ka3)\ \ for\ k \geq 0$

**3. Fuzzy Assignment Problem**

**3.1.Fuzzy assignment problem**

The Fuzzy Assignment problem is defined as minimization linear programming problem. Also, Fuzzy Assignment problem is expressed in mathematical form:

$$MaxZ = \sum\sum cijxij$$

$$i \in Aj \in T$$

Subject to the constraint:

$$\sum xij = 1, for\ j\ =\ 1, \dots, n\ (2)\ j \in T$$

$$\sum xij = 1, forj = 1, \dots, n\ (3)$$

$$i \in A$$

$$xij \geq 0$$

In the see questions variable *cij* refers to weight or cost of performance related to task and *xij* also indicates the Fuzzy

Assignment of agent *i* to task *j*, taking value 1 if the Fuzzy Assignment is done and 0 otherwise.

This problem can be defined in the form of a table as well. Suppose there are *m* jobs and *n* devices which are available, if the cost of doing jth work by it hperson is *cij*, then all costs regarding to this issue can be demonstrated as a table 1 :

**Table 1 Tabular of Fuzzy Assignment problem**

| Device/Jobs | 1 | 2 | 3 | …................j | …........................m |
|---|---|---|---|---|---|
| 1 | $C_{11}$ | $C_{12}$ | $C_{13}$ | …..................... $c_{1j}$ | …........................... $c_{1m}$ |
| 2 | $C_{21}$ | $C_{22}$ | $C_{23}$ | …..................... $c_{2j}$ | …........................... $c_{2m}$ |
| 3 | $C_{31}$ | $C_{32}$ | $C_{33}$ | …..................... $c_{3j}$ | …........................... $c_{3m}$ |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| n | $C_{n1}$ | $C_{n2}$ | $C_{n3}$ | …..................... $c_{nj}$ | …........................... $c_{nm}$ |

**3.2.Fuzzy Assignment Hungarian Algorithm:**

**Steps1:** Fuzzy assignment problem is converting the fuzzy cost to crisp cost value.

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**Step 2: Hungarian Algorithm**

**3.3 Numerical Example:**

Consider the problem of assigning four machines that are available to do four various jobs. The following matrix gives the cost in rupees of producing jobs $i$ on machine $j$ and is represented by triangular fuzzy numbers.

**Table 2 Assign the machines to different jobs so that the total cost is minimized**

| Cij | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | (1,5,9) | (3,7,11) | (7,11,15) | (2,6,10) |
| $J_2$ | (4,8,12) | (1,5,9) | (4,9,13) | (2.6.10) |
| $J_3$ | (0,4,8) | (3,7,11) | (6,10,14) | (3,7,11) |
| $J_4$ | (6,10,14) | (0,4,8) | (4,8,12) | (-1,3,7) |

**Solution:**

**Step1:** Now we convert the fuzzy cost to the crisp cost value by applying Robust ranking method

The membership function of the triangular fuzzy number (1,5,9) is

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{(x-a)}{(5-4)} & 1 \le x \le 5 \\ 1 & x = 5 \\ \frac{(x-a)}{(9-5)} & 5 \le x \le 9 \end{cases}$$

**Using Ranking Method**

$$\boldsymbol{M0 = \frac{1}{4}((2a2 + a1 + a3) = \frac{1}{4}(2(5) + 1 + 9) = 5}$$

$$\boldsymbol{M0 = \frac{1}{4}((2a2 + a1 + a3) = \frac{1}{4}(2(7) + 3 + 11) = 7}$$

**Table 3 Using Ranking Method**

| | | | |
|---|---|---|---|
| (1,5,9)=5 | (3,7,11)=7 | (7,11,15)=11 | (2,6,10)=6 |
| (4,8,12)=8 | (1,5,9)=5 | (4,9,13)=9 | (2.6.10)=6 |
| (0,4,8)=4 | (3,7,11)=7 | (6,10,14)=10 | (3,7,11) )=7 |
| (6,10,14)=10 | (0,4,8)=4 | (4,8,12)=8 | (-1,3,7)=3 |

**Table 4. New Cost Table**

| Cij | M1 | M2 | M3 | M4 |
|---|---|---|---|---|
| J1 | 5 | 7 | 11 | 6 |
| J2 | 8 | 5 | 9 | 6 |

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| | | | | |
|---|---|---|---|---|
| J3 | 4 | 7 | 10 | 7 |
| J4 | 10 | 4 | 8 | 3 |

**Step 2: Hungarian Algorithm**

Subtract minimum value of each row from corresponding row values. So, we have result matrix

| | | | |
|---|---|---|---|
| 0 | 2 | 6 | 1 |
| 3 | 0 | 3 | 1 |
| 0 | 3 | 6 | 3 |
| 7 | 1 | 5 | 0 |

**Figure 1 : Subtract minimum value of each row from corresponding row values**

Subtract maximum value of each column from corresponding row values. So, we have the result matrix:

| | | | |
|---|---|---|---|
| 0 | 2 | 3 | 1 |
| 3 | 0 | 0 | 1 |
| 0 | 3 | 3 | 3 |
| 7 | 1 | 2 | 0 |

**Figure 2 : Subtract maximum value of each row from corresponding row values**

Draw line across rows and columns in which all zeros are covered with minimum number of lines. The result matrix is:

| | | | |
|---|---|---|---|
| 0 | 2 | 3 | 1 |
| 3 | 0 | 0 | 1 |
| 0 | 3 | 3 | 3 |
| 7 | 1 | 2 | 0 |

**Figure 3 : Line across rows and columns**

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 1 |
| 4 | 0 | 0 | 2 |
| 0 | 2 | 2 | 3 |
| 7 | 0 | 1 | 0 |

**Figure 4 If the number of lines is smaller n.**

| | | | |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 5 | 0 | | 2 |
| 0 | 1 | 1 | 2 |
| 8 | 0 | 1 | |

**Figure 5 Minimum Z=7+9+4+3=23**

### 3.4. Fuzzy Assignment Problem Using Branch and Bound method

The Assignment Problem of n×n cost matrix of real numbers is as follows. Consequently, the general assignment problem has a mathematical form.

$$Max\ Z = \sum \sum cijxij$$

$$i \in 1 j \in 1$$

Subject to the constraint:

$$\sum xi = 1$$

$$i = 1$$

$$\sum xj = 1$$

$$j = 1$$

$$xij = \begin{Bmatrix} 1 \text{ if jth job is askigned to the } i^{\text{th}} \text{ person} \\ 0 \text{ Otherwise} \end{Bmatrix}$$

**3.5. Numerical Example:**

Consider the problem of assigning four machines that are available to do four various jobs. The following matrix gives the cost in rupees of producing jobs $i$ on machine j and is represented by triangular fuzzy numbers.

**Table 5. Cost matrix of job assignment problem.**

| Cij | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | (1,5,9) | (3,7,11) | (7,11,15) | (2,6,10) |
| $J_2$ | (4,8,12) | (1,5,9) | (4,9,13) | (2.6.10) |
| $J_3$ | (0,4,8) | (3,7,11) | (6,10,14) | (3,7,11) |
| $J_4$ | (6,10,14) | (0,4,8) | (4,8,12) | (-1,3,7) |

Assign the machines to different jobs so that the total cost is minimized.

**Solution:**

**Step 1:**Fuzzy assignment problem is convert the fuzzy cost to crisp cost value.

**Table 5. Fuzzy job assignment problem convert the fuzzy**

| Cij | M1 | M2 | M3 | M4 |
|---|---|---|---|---|
| J1 | 5 | 7 | 11 | 6 |
| J2 | 8 | 5 | 9 | 6 |
| J3 | 4 | 7 | 10 | 7 |
| J4 | 10 | 4 | 8 | 3 |

**Step: Branch and Bound Method**

Selecting the row minimum and adding up all the values

$$P111 = 5 + 5 + 4 + 3 = 17, P121 = 7 + 6 + 4 + 3 = 20, P131 = 13 = 11 + 5 + 4 + 3 = 23, P141 = 6 + 5 + 4 + 4 = 19$$

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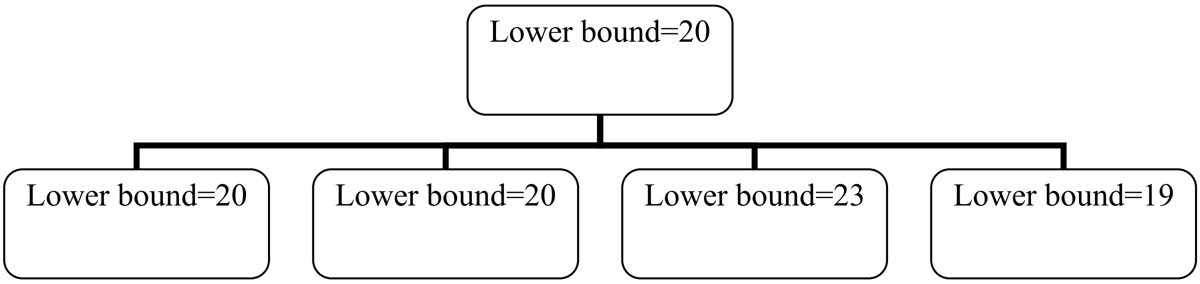


**Figure 6 Lower bound cost for assigning job to subjects**

| | | | |
|---|---|---|---|
| 5 | 7 | 11 | 6 |
| 8 | 5 | 9 | 6 |
| 4 | 7 | 10 | 7 |
| 10 | 4 | 8 | 3 |

Candidate A has a lower bound of 19.As a result. A should be assigned to 4 and further branching should begin from here as has been assigned to 4. Next cheek D for 1 or 2 or 3 as a has been assignedto4.4corresponding row and column would be omitted from consideration when we cheek for D. Following the initial branch, the Following position were left for candidate A, B and C.

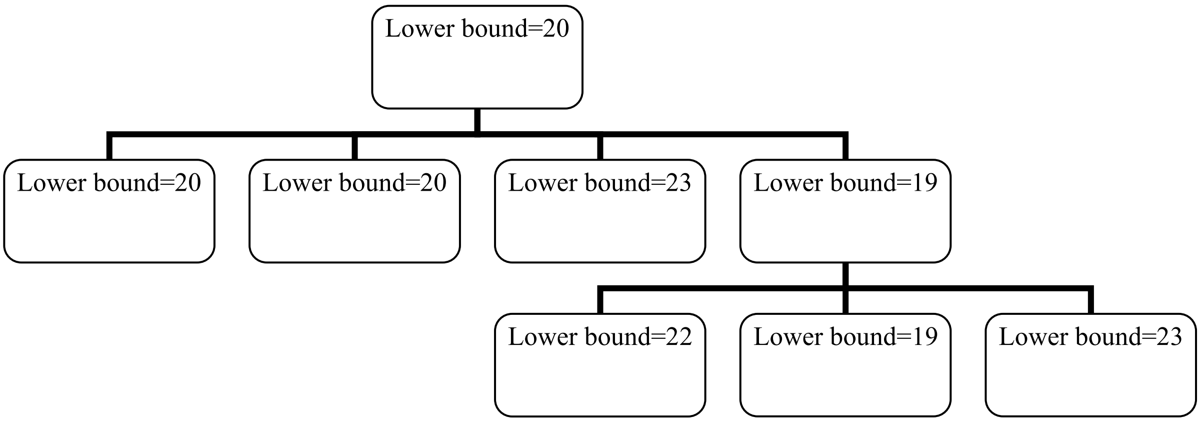


**Figure 7 Lower bound cost for assigning job to subjects 2**

| | | | |
|---|---|---|---|
| 5 | 7 | 11 | 6 |
| 8 | 5 | 9 | 6 |
| 4 | 7 | 10 | 7 |
| 10 | | 8 | 3 |

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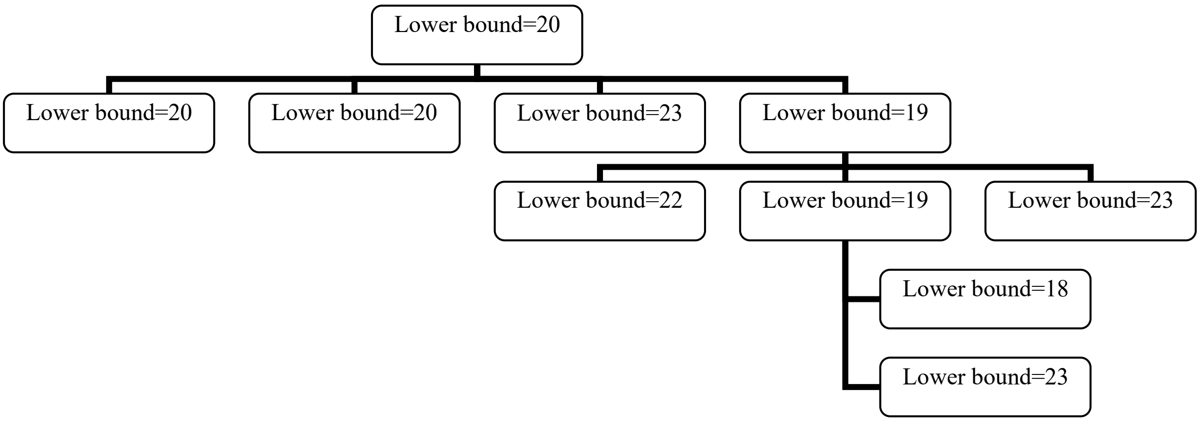


Figure 8 Lower bound cost for assigning job to subjects 3 & 4

Optimal Solution=18

### 4. Discussion

The result indicate that the Branch and Bound Method yields a better optimal solution (Sriniwas, B., & Ganesan, G. (2015). Compared to the Hungarian Method (Kuhn, H. W. (1955). This suggest that the Branch and Bound Method is able to explore the solution space more effectively and find a better assignment of workers to tasks. The Branch and Bound Method uses a more systematic approach to explore the solution space which may lead to better solutions. The Hungarian Method uses a combination optimization strategy, while the Branch and Bound Method uses a free search strategy. The latter is more effective in certain cases. Branch and Bound Method is a more effective optimization technique than Hungarian Method for this particular problem instance. So, Branch and Bound Method is better than Hungarian method.

### 5. Conclusion

This paper presents an approach to solve both the assignment problem and FUZZY assignment problem using Hungarian methods and branch and bound method. Branch and bound method is more effective optimization techniques than Hungarian method for this particular problem. Therefore, the branch and bound method is better than the Hungarian method.

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